Modelling Multi-Agent Robot Systems Using Control and Systems Theory

PhD Course, KTH 2013

Lecturer: Andrzej Pronobis

Goals of This Lecture

- Introductory lecture to PhD-level course
- Give you a good intuition of ***Multi-Agent Systems*** (***MAS***) modeling and control
  - The essential theoretical tools for MAS
  - How to implement and simulate MAS
  - How to solve real-world multi-robots problems
- Boost your interest in MAS
- We will solve and implement two control problems. Code: www.pronobis.pro/mas
- Expand on details in future lectures

Course Materials

www.pronobis.pro/mas

Modelling Multi-Agent Robot Systems Using Control and Systems Theory

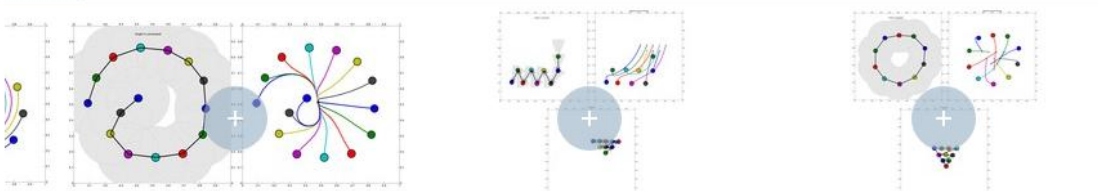
INTRODUCTION

The course covers intermediate to advanced topics in the area of modeling and controlling multi-agent robot systems. The aim is to give you a good intuition of modeling MAS from theoretical problems to practical applications and teach you how design and implement control strategies for multi-agent systems in Matlab.

COURSE COORDINATOR

Andrzej Pronobis, pronobis@cs.washington.edu

HIGHLIGHTS



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Robots with omnidirectional sensing and a trajectory for the rendezvous problem.

Robots with directional sensing and a trajectory for the formation control problem.

Robots with omnidirectional sensing and a trajectory for the formation control problem.

COURSE MATERIALS

MATLAB examples available! Run them now!

CODE

The following Matlab package (code and data) contains useful tools and implementations of basic systems. It should be used as a basis for the solutions developed during the course.

Matlab Package

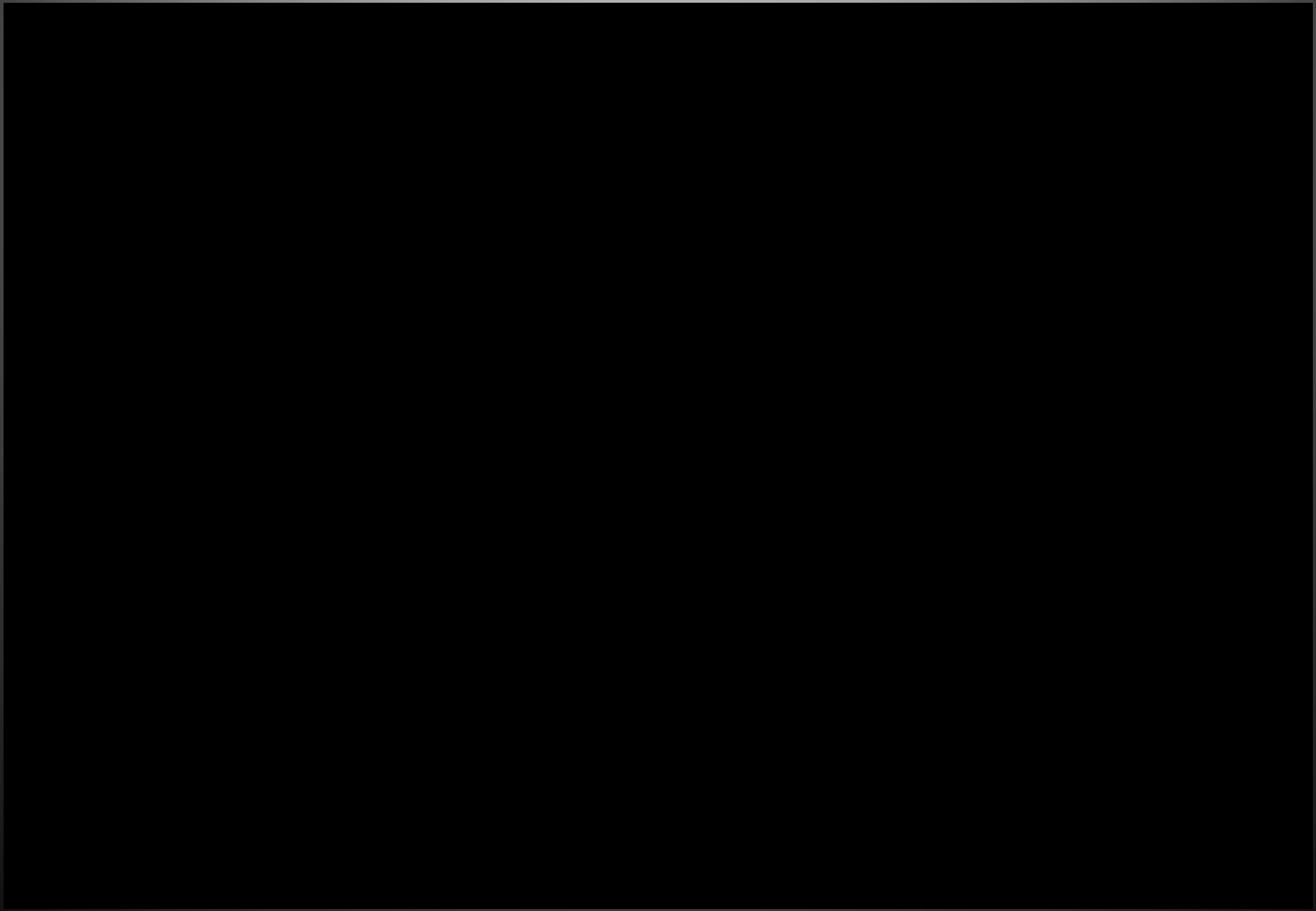
Robot Soccer (RoboCup)

http://www.robocup.org/

Traffic Simulation

http://www.youtube.com/watch?v=azIOCFjDZWA

Image Rendering



http://wwwisg.cs.uni-magdeburg.de/~germer

Modeling Crowd Interactions

http://www.youtube.com/watch?v=IogheZbBcys

Modelling Heavy Metal Mosh Dance…

Collective motion of humans in mosh and circle pits at heavy metal concerts.
(Silverberg, Bierbaum, Sethna, Cohen.) Physical Review Letters, May 2013.

Multi-robot Coordination

http://www.swarm-bots.org/

Outline

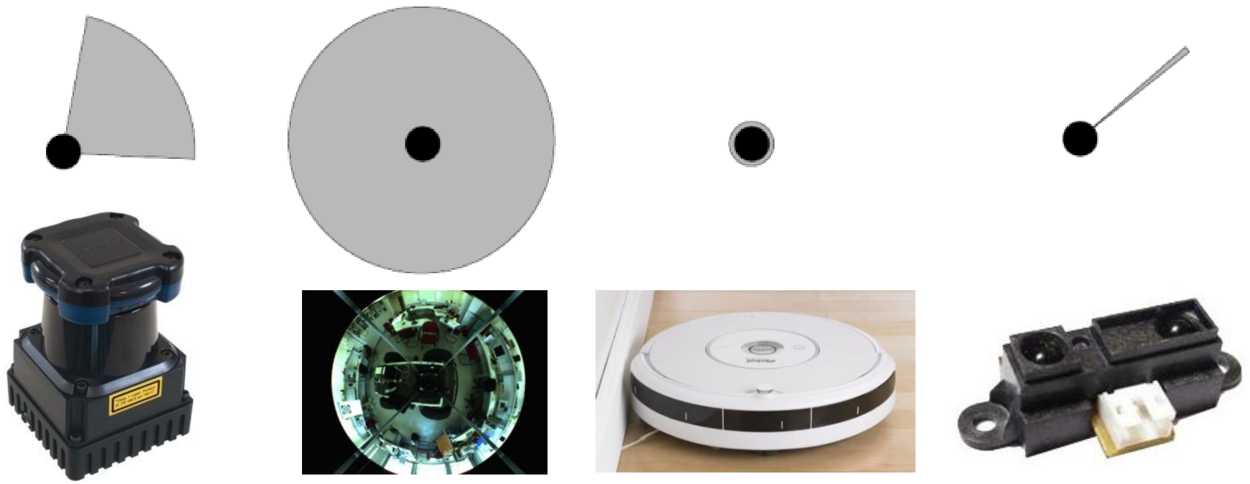
- Intro to Modeling Multi-Agent Systems (MAS)
- Graph Theory for Interaction Graphs
- Agreement Protocol and Rendezvous Problem
  - Simulating in Matlab
- Formation Control Problem
- Summary

Multi-Agent Systems (MAS)

- Multi-agent systems
  - Dynamic units (agents)
    - Sense environment and agents
    - Make decisions
    - Communicate with other agent
  - Signal exchange network
    - Determines how information is exchanged between agents
    - Wireless, visual, chemical signals, sociological interactions
- Multi-agent control
  - How to understand and achieve global system behaviors from local agent behaviors

Networks and Local Interactions

- Networks of local interactions arise due to
  - Locality in sensing



Laser Scanners | Omni Vision | Tactile Sensors | IR Sensors

  - Locality in communication
    - Range - saving energy
    - Bandwidth

Problems for This Lecture

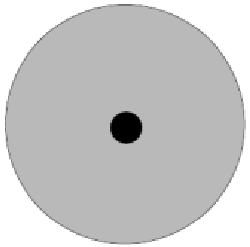
- Agents: mobile robots
- No global map of the environment
  - Example: Rescue scenarios

- Robots only perceive distance to other robots
- Communication through sensing
- Two different robot platforms

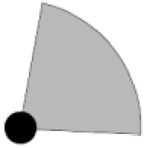
Robot Platforms

Swarmanoid

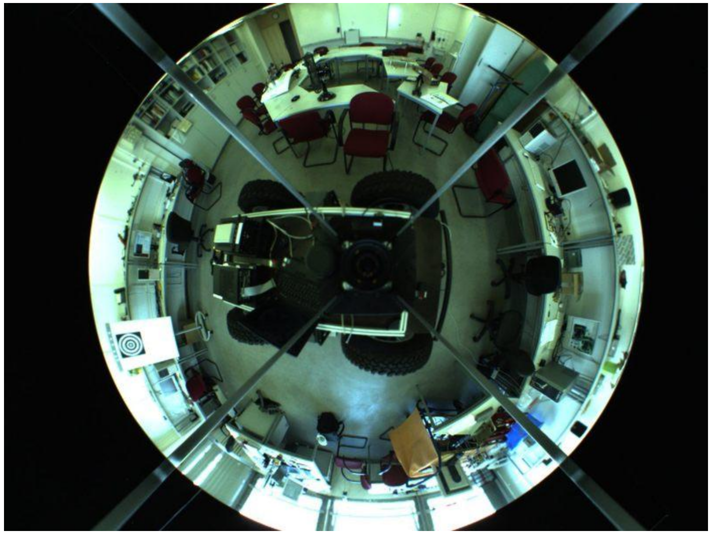
Omni-directional Vision



Directional depth camera (Kinect)

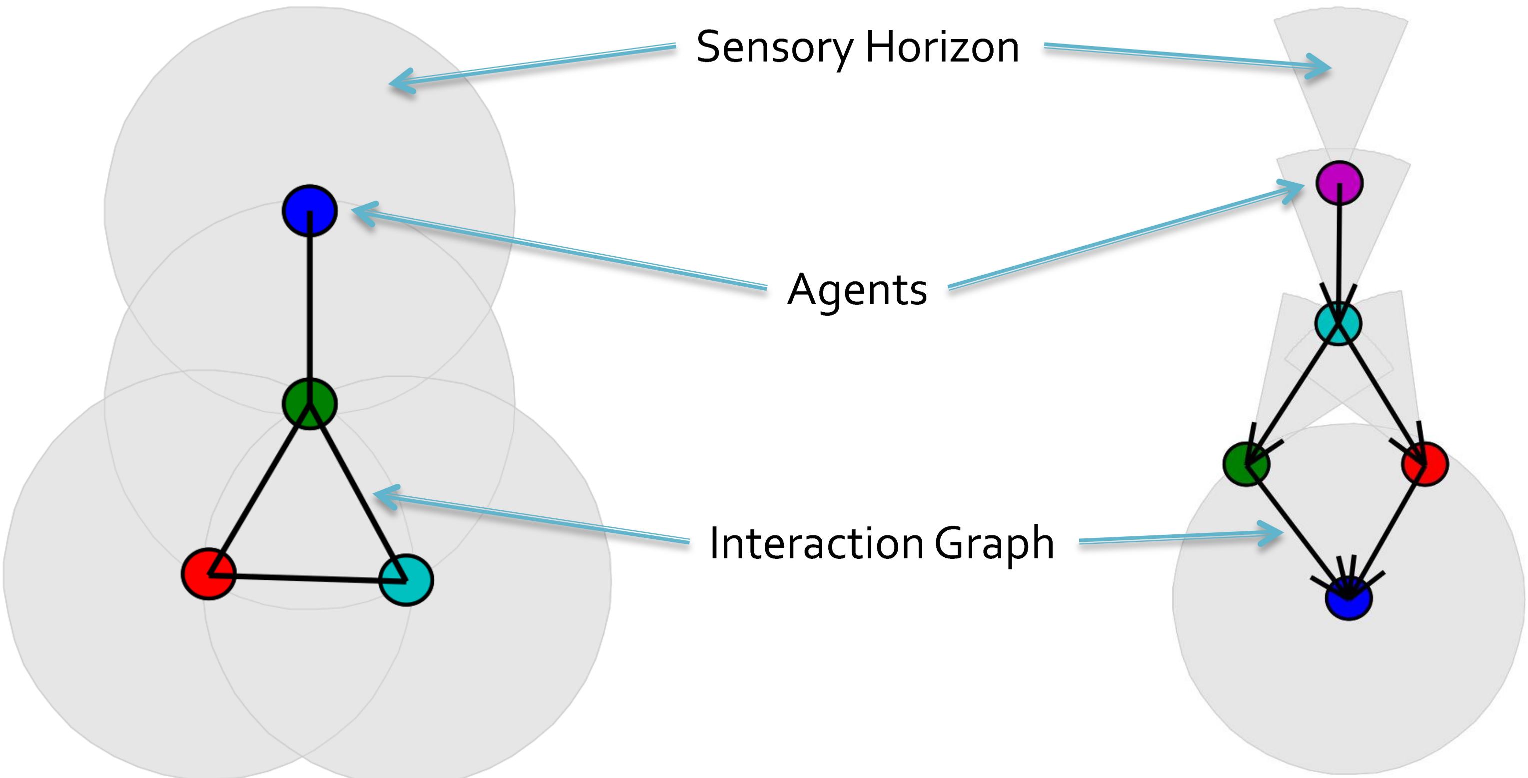


TurtleBot



Graph-based Interaction Models

- Network of agents can be viewed as a graph

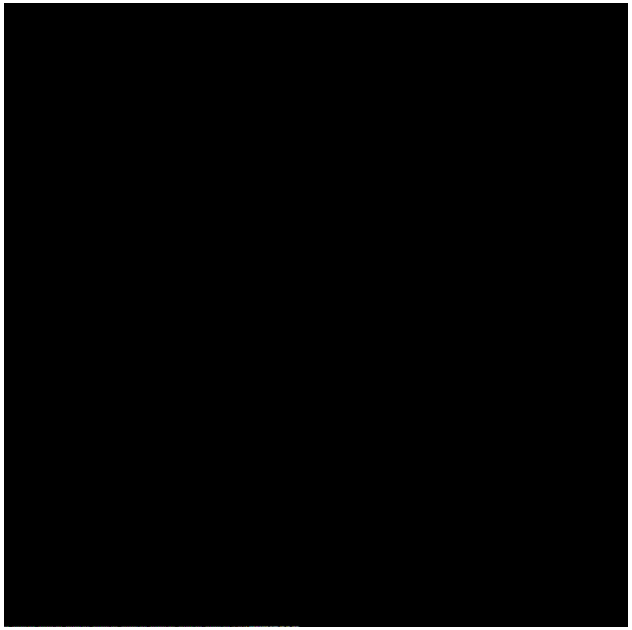


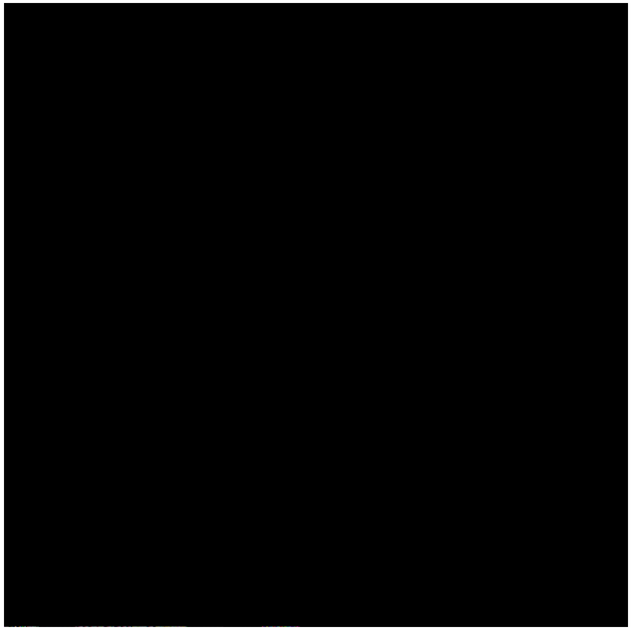
- Omni-directional sensing
- Bi-directional information exchange
- Model: **Undirected graph**

- Constrained field of view
- Unidirectional information exchange
- Model: **Directed graph**

Interaction Protocols

- Several interaction protocols can be formulated and studied theoretically
  - **Agreement (Rendezvous Problem)**
  - **Formation (Formation Control Problem)**
  - Coverage
  - Swarming
  - Distributed Estimation





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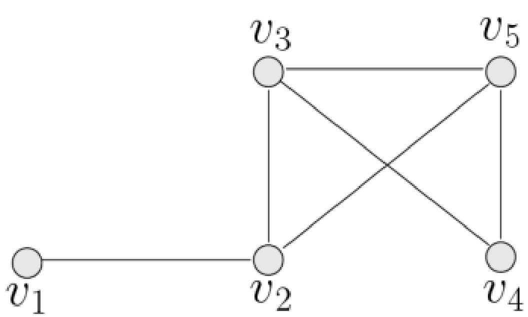
Graph Theory

- Great tool for analyzing networks

Undirected Graphs



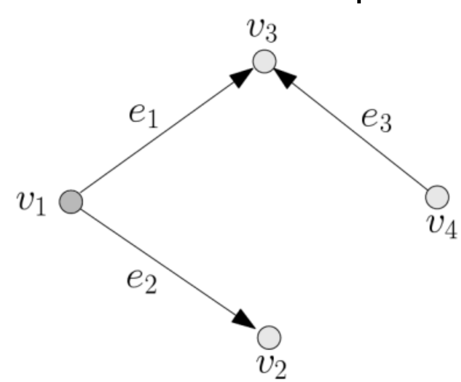
$$\mathcal{G} = (V, E)$$

Set of vertices

Set of edges

Unordered pair $\{v_i, v_j\} \in E \subseteq [V]^2$

Directed Graphs



$$\mathcal{D} = (V, E)$$

Ordered pair $(v_i, v_j) \in E$

Tail

Head

- Neighborhood of a vertex

$$N(i) = \{v_j \in V \mid v_i v_j \in E\}$$

Algebraic Graph Theory

- Adjacency matrix

$$[A(\mathcal{G})]_{ij} = \begin{cases} 1 & \text{if } v_i v_j \in E, \\ 0 & \text{otherwise.} \end{cases} \qquad [A(\mathcal{D})]_{ij} = \begin{cases} 1 & \text{if } (v_j, v_i) \in E(\mathcal{D}) \\ 0 & \text{otherwise,} \end{cases}$$

- Degree matrix (undirected graph)

Degree of vertex $d(v_i)$ represents cardinality of neighborhood set $N(i)$

$$\Delta(\mathcal{G}) = \begin{pmatrix} d(v_1) & 0 & \cdots & 0 \\ 0 & d(v_2) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & d(v_n) \end{pmatrix}$$

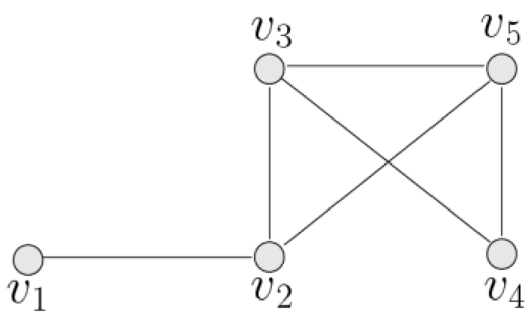
- In-degree matrix (directed graph)

$d(v_i)$ represents the in-degree (counts incoming edges only)

Graph Laplacian

- For undirected graphs

$$L(\mathcal{G}) = \Delta(\mathcal{G}) - A(\mathcal{G})$$



$$L(\mathcal{G}) = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 3 & -1 & 0 & -1 \\ 0 & -1 & 3 & -1 & -1 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & -1 & -1 & -1 & 3 \end{pmatrix}$$

- For directed graphs – In-degree Laplacian

$$L(\mathcal{D}) = \Delta(\mathcal{D}) - A(\mathcal{D})$$

Properties of Laplacian

- Symmetric and positive semi-definite
- Eigenvalues can be ordered as

$$\lambda_1(\mathcal{G}) \leq \lambda_2(\mathcal{G}) \leq \cdots \leq \lambda_n(\mathcal{G})$$

- Smallest eigenvalue is always zero

$$\mathbf{1} \in \mathcal{N}\left(L(\mathcal{D})\right) \quad \lambda_1(\mathcal{G}) = 0$$

- Is the graph connected?
  - If for every pair of vertices there is a path
  - IFF $\lambda_2(\mathcal{G}) > 0$
  - As many connected sub-graphs as zero eigenvalues

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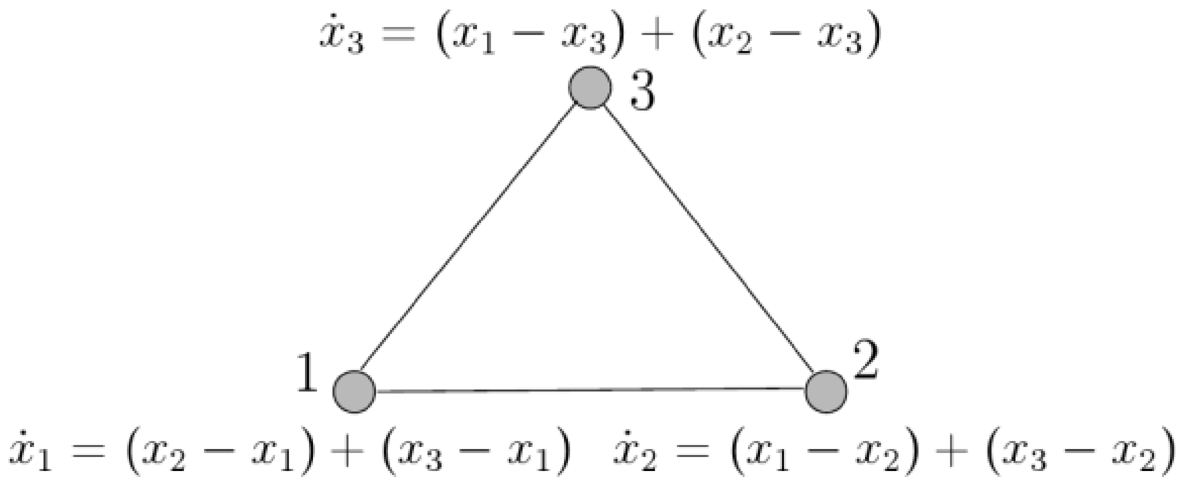
Agreement Protocol

- Agents agree on a value of a parameter
- Definition
  - n dynamic agents
  - Interconnected via relative links
  - Agent's state depends on the sum of its relative states w.r.t. a subset of other agents
- Applications
  - Distributed estimation in sensor networks
  - Flocking/swarming

Rendezvous Problem

- Rendezvous problem
  - Agent's state is its location – mobile robot
  - Agents should meet at one point in space
- Example: agreement protocol over a triangle



- Links pull robots towards each other

State-space Representation

- Continuous-time state-space model

$$\dot{x}_i(t) = \sum_{j \in N(i)} (x_j(t) - x_i(t)), \quad i = 1, \ldots, n$$

$$\boxed{\dot{x}(t) = -L(\mathcal{G})\, x(t)}$$

- For directed graphs: use in-degree Laplacian

$$\boxed{\dot{x}(t) = -L(\mathcal{D}) x(t)}$$

- How can we guarantee convergence?

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State-space Models in Matlab

- Dynamics of a system specified using continuous time-invariant state-space model

$$\begin{aligned}\dot{\mathbf{x}}(t) &= A\mathbf{x}(t) + B\mathbf{u}(t)\\ \mathbf{y}(t) &= C\mathbf{x}(t) + D\mathbf{u}(t)\end{aligned}$$

- Create state-space model

```
sys = ss(A,B,C,D)
```

- In our case:

$$\dot{x}(t) = -L(\mathcal{G})\,x(t) \qquad \dot{x}(t) = -L(\mathcal{D})x(t)$$

A

Simulating

- Simulate
  - Initial condition response

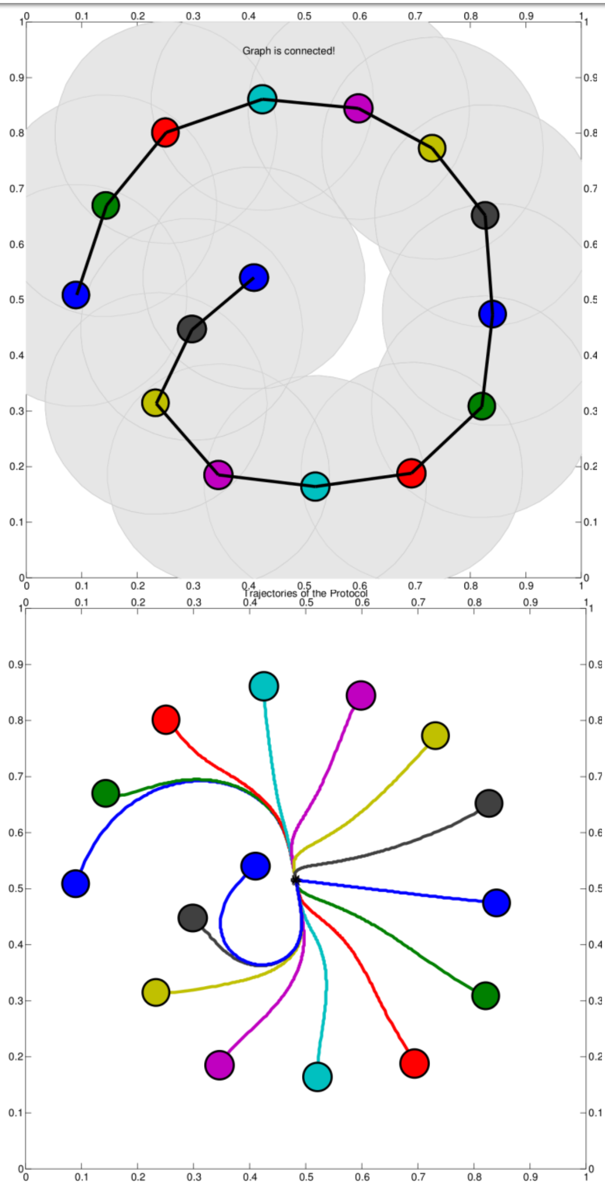
```
[y,t,x] = initial(sys,x0,t)
```

  - Response to arbitrary inputs

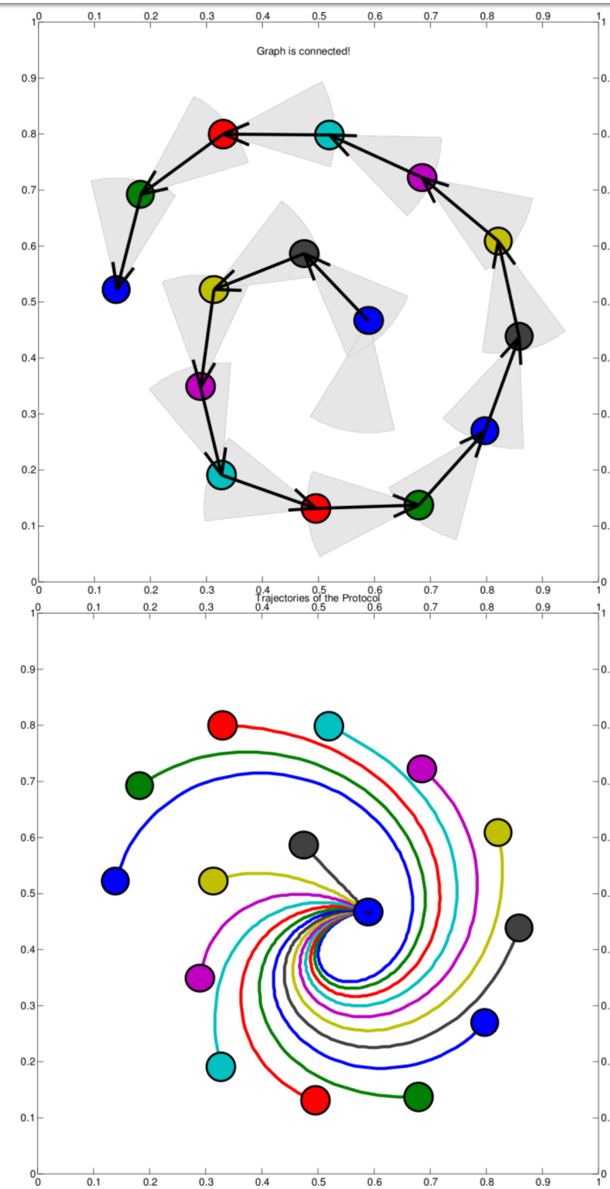
```
[y,t,x] = lsim(sys,u,t,x0)
```

- Basic toolkit for defining and visualizing problems available in course materials

Simulating Trajectories



Run in Matlab



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Formation Control Problem

- Mobile agents move in order to realize a geometrical pattern (formation)
  - Appear often in biological systems (e.g. geese)
- Formations can be specified in several ways
  - Relative state
  - **Shape**
    - Specified in terms of points

$$\Xi = \{\xi_1, \ldots, \xi_n\},\ \xi_i \in \mathbf{R}^p,\ i = 1, \ldots, n,$$

    - Translationally invariant

$$x_i = \xi_i + \tau$$

State-space Representation

- Let's define τ_i as displacement from target

$$\tau_i(t) = x_i(t) - \xi_i, \; i = 1, \ldots, n$$

- Now, apply the agreement protocol to τ_i

$$\dot{\tau}_i(t) = - \sum_{j \in N_f(i)} (\tau_i(t) - \tau_j(t))$$

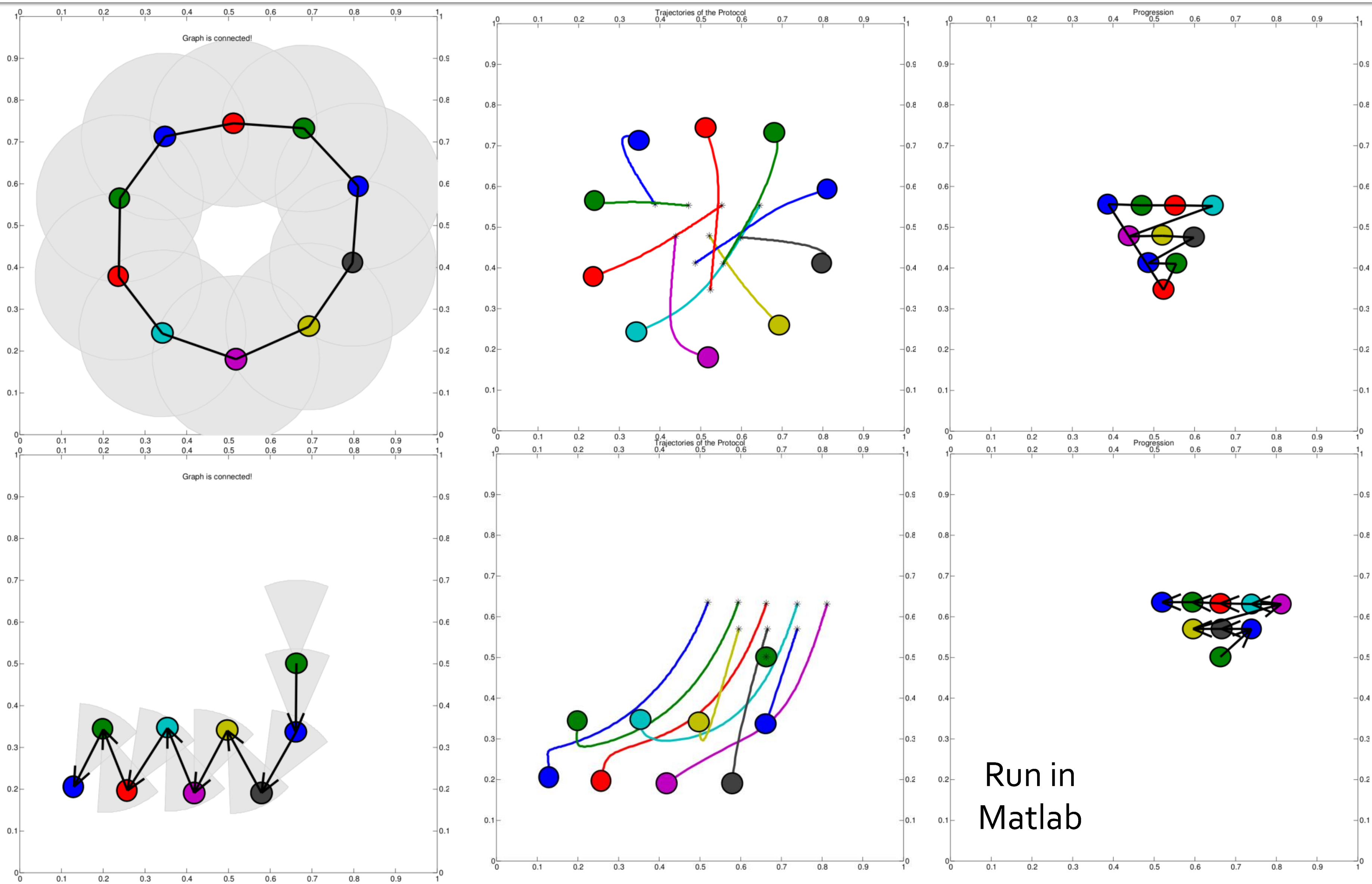
- Since $\dot{\tau}_i(t) = \dot{x}_i(t)$, $\tau_i(t) - \tau_j(t) = x_i(t) - x_j(t) - (\xi_i - \xi_j)$:

$$\dot{x}_i(t) = - \sum_{j \in N_f(i)} (x_i(t) - x_j(t)) - (\xi_i - \xi_j)$$

$$\dot{x}(t) = -L(\mathcal{G})\,x(t) + L(\mathcal{G})\,\Xi$$

Analogous for directed graphs

Simulating Trajectories



Run in Matlab

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Summary

- Intuition about what multi-agent systems are and how to solve control problems
  - From theory to simulations
- Multiple applications in robotics
  - When no global maps and central coordination
- What's next?
  - Dynamic and random networks
  - Switching between formations and control problems
  - Networks as systems (with inputs & outputs)
- Try this at home!

Questions?