

# Novelty Detection Using Graphical Models for Semantic Room Classification

André Susano Pinto<sup>1,2</sup>, Andrzej Pronobis<sup>2</sup>, and Luis Paulo Reis<sup>1,3</sup>

<sup>1</sup> Dep. Informatics Engineering, Faculty of Engineering of the University of Porto

<sup>2</sup> Centre for Autonomous Systems, The Royal Institute of Technology (KTH)  
SE100-44 Stockholm, Sweden

<sup>3</sup> LIACC - Artificial Intelligence And Computer Science Lab. - University of Porto  
Rua Dr. Roberto Frias, s/n 4200-465 Porto, Portugal

**Abstract.** This paper presents an approach to the problem of novelty detection in the context of semantic room categorization. The ability to assign semantic labels to areas in the environment is crucial for autonomous agents aiming to perform complex human-like tasks and human interaction. However, in order to be robust and naturally learn the semantics from the human user, the agent must be able to identify gaps in its own knowledge. To this end, we propose a method based on graphical models to identify novel input which does not match any of the previously learnt semantic descriptions. The method employs a novelty threshold defined in terms of conditional and unconditional probabilities. The novelty threshold is then optimized using an unconditional probability density model trained from unlabelled data.

**Keywords:** Novelty detection, semantic data, probabilistic graphical models, room classification, indoor environments, robotics, multi-modal classification.

## 1 Introduction

There has been several efforts in the areas of artificial intelligence and robotics in creating robots that are able to interact with humans and their environments. One of the important aspects is to endow those robots with a deeper understanding of human environments, not just for the purpose of navigation and obstacle avoidance, but also in terms of human semantics and functionality. An important problem in creating reliable representations of space for robots that are to be deployed in new and unknown realistic environments is to be able to automatically identify gaps in robot's knowledge and act in order to fill those gaps.

This article addresses the problem of *novelty detection* within the context of semantic mapping i.e. generating maps containing *semantic information* about indoor environments, such as homes or offices. In that context, the ability to detect that the observations result from a semantic concept unknown to the robot, and cannot be explained by one of its models, is crucial for generating fully

autonomous and reliable behavior. In order to be robust, the robot must identify novel concepts and instead of making a costly error, refrain from the decision and initiate learning. In particular, we address the problem of novelty detection for room categorization i.e. detecting whether the area in the environment identified as a separate room can be assigned to one of the semantic labels that the robot knows (e.g. *a kitchen* or *an office*) or belongs to an unknown semantic category.

The novelty detection algorithm is implemented on a cognitive robot Dora the Explorer, which already uses a developed architecture based on probabilistic graphical models oriented towards dealing with and reasoning about uncertain semantic information [1]. One of the major problems with using the probabilistic models of the environments for novelty detection is selecting the optimal threshold above which the test sample is considered novel. This problem becomes more difficult when, as in case of Dora, the representation grows as the robot explores the environment. Methods are required to find the right threshold given the current structure of the model which constantly changes. To this end, this work studies methods for novelty threshold selection using probabilistic graphical models.

The rest of this paper is structured as follows. First, we briefly review the related work related to room categorization and novelty detection (Section 2). Then, we give an outline of the architecture of the Dora system and the structure of the conceptual map representing the semantic information (Section 3). Next, we discuss the methods for novelty detection (Section 4) and present results of our preliminary experiments (Section 5). This paper concludes with a summary in Section 6.

## 2 Related Work

The problem of room categorization based on visual information was first addressed in the computer vision community. The research focused mainly on the problem of classifying single images captured in indoor or outdoor environments (scene classification) [2,3]. At the same time, robotics researchers initially employed the 2D laser range sensor being much more robust to variations occurring in the environment and much easier to handle computationally in real time [4].

Multi-modal approaches, such as combining semantic data extracted from several sources or classifiers are expected to have better performance on scene recognition than single-cue approaches. Quattoni and Torralba [5] showed that most scene recognition models work poorly in indoor scenes when compared to outdoor scenes since the properties that characterize rooms changes depending on the category. For instance corridors are well described by global properties and bookstores are well described by the presence of specific objects (books). Galindo et al. [6] also exploits this by defining a bidirectional relation between object and room category, where object defines a room category and a room category provides information on where objects may be found.

Probabilistic representations have been frequently used for spatial modelling in robots operating in the real-world [7,8]. Boutell et al. [9] have studied outdoor

scene classification using *factor graphs* and modelling spatial relations between objects in the scene to extract better knowledge from semantic (high-level) features.

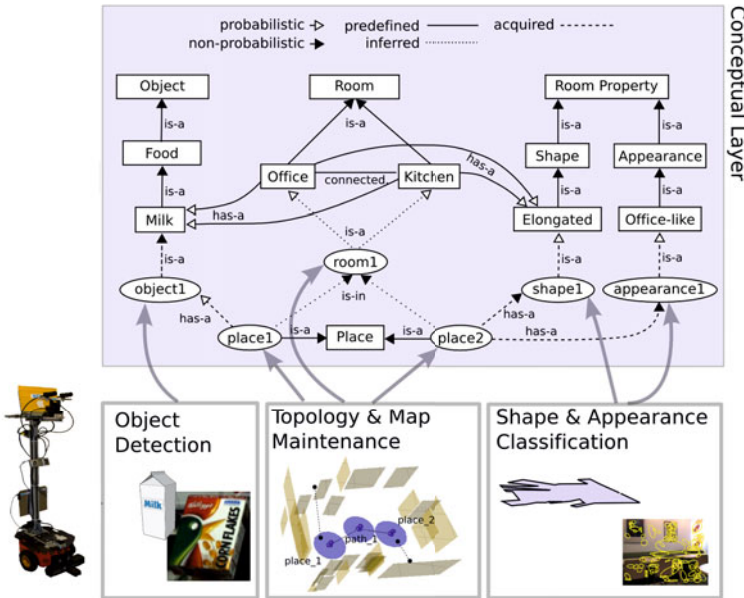
Although our approach is presented in the context of mobile robotics it relies on standard concepts and techniques such as semantic data and graphical models. Those are often used in the area of information retrieval. An interesting example is the usage of an hidden concept layer between visual features and text information to provide automatic image annotation [10].

Novelty detection has been studied for many years and there are several approaches based on statistical analysis [11]. Graphical models have been used to learn distributions of variables, both in supervised and unsupervised ways and by using thresholds on those distributions based solely on the conditional probability, as seen on Bishop [12], a novelty system can be trivially implemented.

However to the knowledge of the authors there is no reference on how to perform novelty detection using graphs that are dynamically generated.

### 3 Dora Architecture Overview

The Dora system [1] consists of several co-operating sub-systems, all of which actively use or maintain the spatial knowledge representation (see Figure 1). Only the *conceptual layer* of the representation is of interest to this article. Its role is to aggregate the following semantic information coming from other sub-systems:



**Fig. 1.** Interaction between the sub-systems of Dora with special focus on the conceptual layer

**Doorway detection** is used to segment the continuous space into rooms and map connectivity between them.

**Room size and shape** are obtained by classifying 2D laser scans from laser range finder mounted on the robot and are used as properties of a room. The system utilizes pre-trained set of classifiers to extract rooms sizes (either large, medium or small) and shapes (rectangular or elongated).

**Object detection** is performed in images acquired by the robot through its camera. The system keeps track of the number of objects of each type in each room. Objects are detected by running a pre-trained set of detectors for the following object types: book, cereal box, computer, robot, stapler, toilet paper.

**Room appearance** is categorized from the visual input by using global visual features and a pre-trained set of 7 different models.

As Dora moves through the environment its *conceptual layer* builds a structural and probabilistic representation of space instantiated as a *graphical model*. It includes taxonomy of human-compatible spatial concepts which are linked to the sensed instances of these concepts drawn from lower layers. It is the conceptual layer which contains the information that kitchens commonly contain cereal boxes and have certain general appearance and allows the robot to infer that the cornflakes box in front of the robot makes it more likely that the current room is a kitchen. The conceptual layer is described in terms of a probabilistic ontology defining spatial concepts and linking those concepts to instances of spatial entities (see the example of the ontology in Figure 1).

### 3.1 Conceptual Map

Based on this design, a *chain graph* [13] model is proposed as a representation for performing inferences on the knowledge represented in the conceptual layer. Chain graphs are probabilistic graphical models that combine the properties of both Bayesian Networks and Random Markov Fields. This results in an efficient approach to probabilistic modeling and reasoning about conceptual knowledge.

An exemplary chain graph corresponding to the conceptual map ontology is presented in Figure 2. Each discrete place identified in the environment is represented by a set of random variables, one for each class of relation linked to that place. These are each connected to a random variable over the categories of rooms, representing the “is-a” relation between rooms and their categories. Moreover, the room category variables are connected by undirected links to one another according to the topological map. The remaining variables represent: shape and appearance properties of space as observed from each place, and the presence of objects. These are connected to observations of features extracted directly from the sensory input. Finally, the distributions  $p_s(\cdot|\cdot)$ ,  $p_a(\cdot|\cdot)$ ,  $p_{o_i}(\cdot|\cdot)$  represent the common sense knowledge about shape, appearance, and object co-occurrence, respectively. They allow for inference about other properties and room categories e.g. that the room is likely to be a kitchen, because you are likely to have observed cornflakes in it.

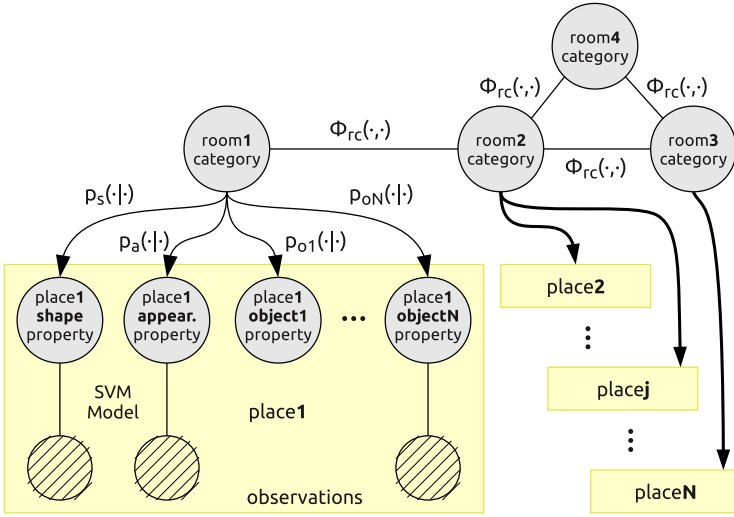


Fig. 2. Example of a chain-graph produced by the *conceptual layer*

The use of graphical models to describe distributions of variables has useful properties. First, they permit inference about uncertain conceptual knowledge. At the same time, they are generative models and therefore allow to calculate the probability on any given subset of variables of the graph, allowing the system to work even when some information is missing.

### 3.2 Factor Graphs

Although the conceptual layer works with *chain graphs*, those can be converted into *factor graphs* [14]. Factor graphs are used throughout this paper as they provide an easier manipulation due to factorization. Moreover, there exist efficient implementations of inference engines operating on factor graph representations [15]. Describing the distribution function in terms of graphs allows to use those engines to efficiently calculate marginals over any given subset of variables by exploiting conditional independence between variables.

A *factor graph* is a bipartite graph connecting two sets of nodes  $X_G$  and  $F_G$  representing random variables and factors. Each factor is described by a function  $\phi$  dependent only on the variables  $x_\phi$  to which the factor is connected. Thus, a factor graph can be seen as a description of probability density function obtained by a product of all the factors. In order to represent the probability, a normalization factor needs to be introduced, resulting in the following equation:

$$P_G(x) = \frac{1}{Z} \prod_{\phi \in F_G} \phi(x_\phi), \quad Z = \sum_{X_G} \prod_{\phi \in F_G} \phi(x_\phi) \quad (1)$$

## 4 Novelty Detection

The aim of novelty detection is to identify data samples originating from a distribution different than one of those the system knows about [11]. It is harder than classification as only positive samples of the class are available rendering normal classification methods unusable. Adding novelty detection capabilities allows to increase reliability of the system. Novelty signal can be used to inform the system that it should proceed with caution as its knowledge does not correctly describe the environment.

Due to the nature of the sensed data which are noisy and uncertain, novelty ought to be treated in a probabilistic fashion. In such case, each sample is associated with certain probability of being generated by a class not known to the agent and a complementary probability  $P(\overline{novel}|x)$  of being generated by a known class. The true positive and false positive rate of a novelty detection system which classifies the set  $N$  of samples as novel is given by:

$$P(\text{true positive}) = \sum_{x \in N} P(novel|x)P(x) \tag{2}$$

$$P(\text{false positive}) = \sum_{x \in N} P(\overline{novel}|x)P(x) \tag{3}$$

It follows that by extending the set  $N$  with new samples, the true positive and false positive rate can never be decreased, leading to the problem where in order to increase detection, the system needs to increase its error. This describes the base of the *error and rejection tradeoff* [16], which states that a system aiming at increasing the true-positive probability will eventually increase its false-positive error.

This way an optimal detector can be formulated by achieving the maximum true-positive probability without its false-positive probability increasing beyond a given limit. This is equivalent to the *continuous knapsack problem* which allows for a greedy solution by sorting the items with a value per weight function. In the case of a detection system, this can be defined as:

$$value(x) = P(novel|x) \tag{4}$$

$$weight(x) = P(\overline{novel}|x) \tag{5}$$

$$cost(x) = value(x)/weight(x) \tag{6}$$

$$= \frac{P(novel|x)P(x)}{P(\overline{novel}|x)P(x)} \tag{7}$$

Therefore, a novelty detection system before classifying a sample  $a$  as novel, should (greedily) classify any sample  $b$  with a smaller cost as that would achieve a higher true positive probability given a fixed false positive one.

$$\frac{P(novel|b)}{P(\overline{novel}|b)} < \frac{P(novel|a)}{P(\overline{novel}|a)} \tag{8}$$

This relation between  $a$  and  $b$  can further be simplified into:

$$P(\overline{novel}|b) < P(\overline{novel}|a) \quad (9)$$

Based on this, it can be said that a novelty detection system aims at defining an order relation on all the possible inputs equivalent to the order defined by the function:  $P(\overline{novel}|x)$ . Then, the detector can be described by the largest  $P(\overline{novel}|x)$  accepted which is the principle of thresholding.

Using Bayes rule and assuming a constant  $P(\overline{novel})$ , a ratio between conditional and unconditional probabilities of the input  $x$  is obtained. Such a ratio is a suitable function for implementing a novelty detector system with optimal thresholding.

$$P(\overline{novel}|x) = \frac{P(x|\overline{novel})P(\overline{novel})}{P(x)} \propto \frac{P(x|\overline{novel})}{P(x)} \quad (10)$$

Note, however that in the case of dynamic graph structures, an assumption on a constant  $P(\overline{novel})$  is quite strong. Although in our first approach, it is assumed to be constant, the authors acknowledge that structure plays an important role and should be used as prior information when calculating  $P(\overline{novel})$ .

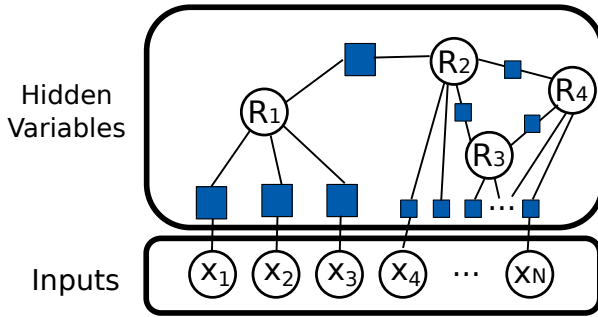
#### 4.1 Conditional Probability

The conditional probability models the distribution of variables given that the agent knowledge holds true and can be used to describe the generating classes of the sensed sample. Under that its natural to model it with a graphical model that combines the learned variables and categories as well with the relations between them.

In Dora case, this is equivalent to use the graphical model used by it to describe its current believes on the variables modelled by the system. That graph is built, by the conceptual layer, by instantiating the information extracted from other layers together with the conceptual knowledge such as: objects are properties of rooms, rooms are connected between each other.

Figure 3 illustrates a graph  $G$  built from the conceptual layer to represent the conditional probability on the sensed variables  $x$ . A set of hidden variables is added to represent the conceptual knowledge the system is aware of. In the presented graph variables  $R_i$  were added to model the room categories that influence the directly sensed features on each physical room, as well connectivity factors between each room.

The factors connecting the variables are trained by the system by searching databases of common knowledge to build potentials describing how likely it is that a specific set of values of certain variable types is likely to occur. For instance: it is very likely to find a cereal box in a kitchen; and it is unlikely to find bathroom connected to another bathroom. We propose using such a graph  $G$  built by the conceptual layer for modelling  $P_G(x)$  as an approximation for  $P(x|\overline{novel})$ .



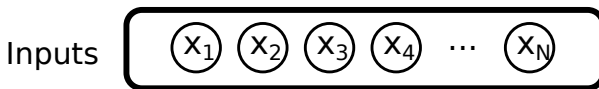
**Fig. 3.** Illustration of a factor graph modelling a distribution of a set  $x$  of sensed variables.

### 4.2 Unconditional Probability

With only access to labelled data a common approach is to define a threshold assuming that  $P(x)$  is constant through all the samples.

Its important to notice that in several cases assuming it to be constant leads to discarding the factor. Nonetheless, here the distributions are dynamically growing as the system learns more on the environment. So the normalizing argument  $P(x)$  has to be evaluated for each new subset of  $x$ .

Assuming that the unconditional distributions generates all possible outcome with the same probability we can model it with  $\prod 1/\#x_i$ , where  $\#x_i$  denotes the cardinality of the state space of variable  $x_i$ . In graphical model terms this is represented to a factor graph  $U$  with the variables but without any factors as illustrated on Figure 4.



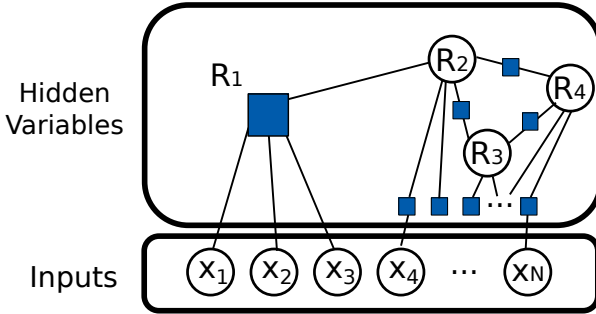
**Fig. 4.** Without any existing factors, this graph  $U$  represents a uniform distribution over any set of its variables

Having a graphical model  $G$  built to model the known data distribution and a model  $U$  for the unconditional probability a novelty threshold would be given by:  $P_G(x)/P_U(x)$ . Here  $P_U(x)$  can be seen as a normalizing factor to lever all the  $P_G(x)$  on any set of variables  $x$  into the same measure units (error rate), such that a static threshold can be implemented. For example the conditional probability would yield very small values on large sets of variables  $x$  than in small sets due to the spreading over the dimensions of the sample space. A novelty measure is seen as a ratio on how much introducing the known concepts helps to understand the observed result.



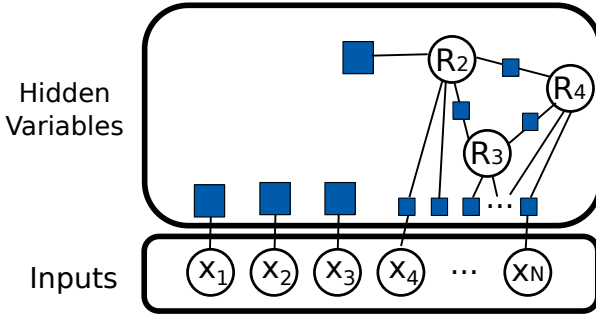
### 4.3 Semi Supervised: Using Unlabelled Data

Nonetheless, its often the case that there is access to extra data that allows to obtain a better approximation to the unconditional probability than the uniform one. In specific, all the knowledge of the agent can be considered to hold true apart from complete knowledge on the categories of a room. In that case a single big factor can be used to model all the variables directly dependent on the possibly novel room as illustrated in Figure 5.



**Fig. 5.** Without being able to model variable  $R_1$  all the variables directly dependent on it become dependent between each other introducing a single big factor

For practical reasons, it is impossible to train such a factor, and simplifications need to be performed. Here, it was assumed that it can be approximated by factorizing it in several single factors such that all variables become independent. Additionally those single factors can easily be trained by using unlabelled data. Obtaining this way a graphical model  $I$ , illustrated in Figure 6, to be used as approximation for the unconditional probability



**Fig. 6.** By factorizing the single factor introduced by room 1 not being necessarily known, several single factors are obtained that can be trained from unlabelled data

Once again the novelty threshold would be given by  $P_G(x)/P_I(x)$ . This time the addition of the unconditional factors can be understood as an attempt to compensate for an existing bias on the unconditional distribution. This is an important step to achieve a correct order-relation of the inputs sample for implementing a novelty threshold.

## 5 Results

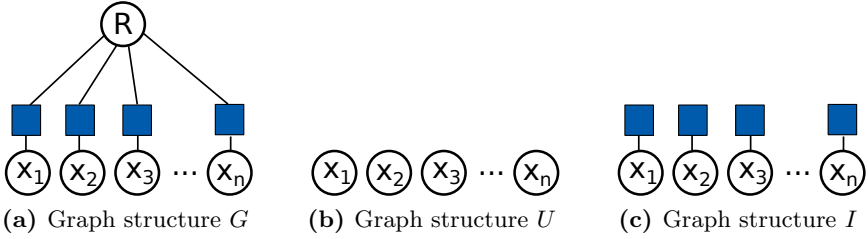
In order to verify the performance of the proposed threshold functions, a synthetic dataset was generated. As the point on this initial work was only to test the correctness of the presented threshold function and approximation capabilities by using a uniform or an independent model, only information regarding direct features of a room were modelled and no structured knowledge such as room connectivity was taken in account.

The synthetic distribution assumes that an independent and variable number of features  $x$  is generated by a given room category. In whole there was 11 different room categories and 9 different measured feature types. The number of sensed features is dynamic and mimic the type of information extracted when running on the robot. Due to that it is possible that on a given sample a certain feature type can be present more than once or not be present at all (e.g.: room shape is extracted from 2D laser scans in more than one position in the room, and information about detected objects is only present when the robot previously tried to detect objects on a given room).

The sensed properties and room categories were chosen to mimic as close as possible the reality and they are based on a previously built ontology from web data. There is in total 11 different room categories ranging from: corridor, hallway, 1 person office, 2 person office, bathroom, conference hall, etc. . . , and there is 9 different extracted features: room size, room shape, room appearance and 6 different objects (e.g. book, cereal box, computer).

From the distribution, 100 labelled samples for 5 of the 11 room categories were drawn to represent the known categories and 1000 unlabelled samples were drawn from all the room categories for learning the unconditional probability distribution. Using those samples, factors were learnt for the graphs used to model the conditional distribution and the independent unconditional distribution. Figure 7 shows the graph structure used for approximate the trained conditional and unconditional distributions. Its important to notice that graph  $G$  used to model the known classes when given enough labelled data is able to exactly learn the conditional distribution as it uses the same structure as the created synthetic distribution.

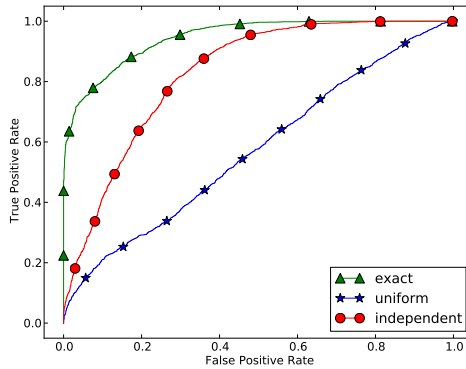
Using the learned models  $G$ ,  $U$  and  $I$ , two thresholds were trained:  $P_G(x)/P_U(x)$  assuming a uniform unconditional distribution and  $P_G(x)/P_I(x)$  assuming an independent unconditional distribution. Since the distribution is synthetic there is access to  $P(x)$  and  $P(x|\overline{novel})$  and a perfect threshold function could also be created to test how far the presented thresholds are from optimal.



**Fig. 7.** The graph structures used to model the conditional and unconditional probability for implementing the novelty thresholds  $P_G(x)/P_U(x)$  and  $P_G(x)/P_I(x)$

### 5.1 Probability Ratio Comparison

First, the performance of the novelty threshold selection was plotted for a set of 1000 samples taken from the whole distribution (Figure 8). The samples were uniformly generated by graphs with 5, 10, 15, 20, 35, 50 features. Additionally the feature types were also uniformly sampled, for that it is possible that in certain samples some feature types were sensed more than once and other were not sensed at all. This was chosen to mimic the dynamic properties expected to see when implemented on a robot.



**Fig. 8.** ROC curve comparing novelty detection performance under samples with variable size of sensed properties

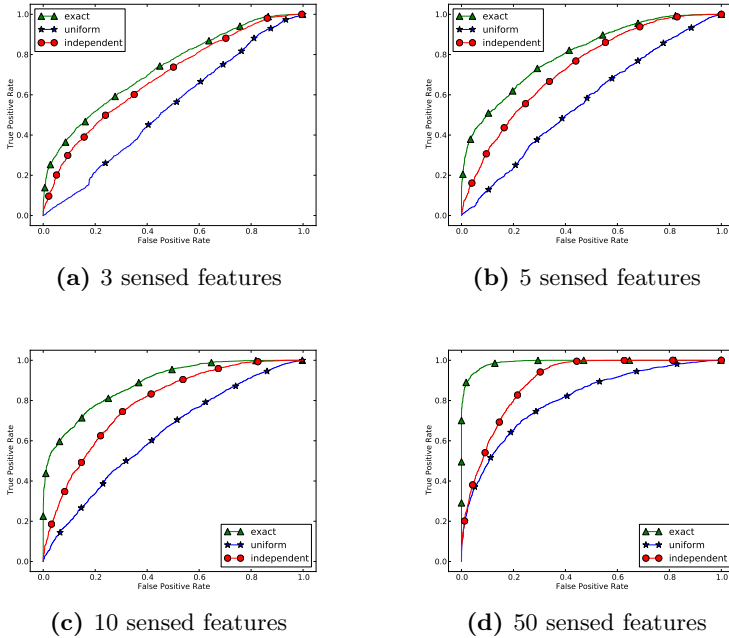
The convex shape of the optimal threshold shows that the ratio between conditional and unconditional probability is indeed a suitable detector for implementing a threshold when the samples are taken from dynamic distributions when  $P(novel)$  is constant (e.g. some samples where there is only access to room size versus samples where there is a lot of information about the room properties).

Its also possible to see how important it is to estimate a correct unconditional probability in order to obtain a correct novelty measure on the inputs. The assumption of a uniform unconditional probability has led to very poor results.

That is probably explained by the semantic properties being highly biased towards some values. This shows that bias plays an important role in detecting whether a given sensed value is a valuable cue about the room category.

### 5.2 Performance Changes with Amount of Available Information

In order to measure the performance impact as more semantic information becomes available, ROC curves were plotted for samples grouped by the number of sensed semantic features.



**Fig. 9.** ROC curves plotted showing performance of the presented novelty detection method for graphs generated for different amount of sensed features

It is possible to see that as the system gains more semantic information, it becomes easier to detect novelty. The size of the input space increases and allows the existing classes to become more easily distinguished.

The performance of the independent threshold decreases as the number of sensed features increases. This is easily explained by the fact that the graph  $I$  is not able to model the existent dependence between the features. This becomes obvious as the number of features increases (e.g. graph  $I$  perfectly models  $P(x)$  in the case where only 1 feature is sensed).

The uniform threshold shows poor performance especially for samples with small amount of features where it performs almost no better than random. The performance increases as the size of sensed features increases but nonetheless is very small when compared to the optimal threshold.

## 6 Conclusions and Future Work

In this paper we presented how to define a stable novelty threshold function on top of *probabilistic graphical models* instantiated dynamically from sensed semantic data. The presented technique is based on the ratio between a conditional and unconditional probability and when perfect information exists it performs an optimal novelty detection under the assumption that  $P(\text{novel})$  is constant across all the graph structures.

It was also shown that a correct estimation of unconditional probability plays an important role specially on small input spaces. Moreover semi-supervised techniques, implemented with the access to unlabelled data, can be used to significantly improve novelty detection performance.

Given the synthetic distribution, an assumption on an uniform distribution has led to very poor results. The same behaviour is expected to in real world distributions based on semantic data. For that reason, and due to easy access to unlabelled data, special attention will be given to using semi-supervised techniques for novelty detection.

After this initial study on how to detect new semantic classes based on graphical models, future work will focus on how to use the structured information available from the conceptual layer to be able to detect which variable of the graph is novel and what makes it different from other previously learned classes. That will lead to generation of useful information that can be used for communication with the user and performing active learning of new room categories.

## References

1. Hanheide, M., Gretton, C., Dearden, R.W., Hawes, N.A., Wyatt, J.L., Pronobis, A., Aydemir, A., Göbelbecker, M., Zender, H.: Exploiting probabilistic knowledge under uncertain sensing for efficient robot behaviour. In: Proceedings of the 22nd International Joint Conference on Artificial Intelligence (IJCAI 2011), Barcelona, Spain (2011)
2. Oliva, A., Torralba, A.: Building the gist of a scene: The role of global image features in recognition. *Progress in Brain Research* 155, 23–36 (2006)
3. Torralba, A.: Contextual priming for object detection. *International Journal of Computer Vision* 53(2), 169–191 (2003)
4. Mozos, O.M., Stachniss, C., Burgard, W.: Supervised learning of places from range data using adaboost. In: Proceedings of the 2005 IEEE International Conference on Robotics and Automation, ICRA 2005, pp. 1730–1735. IEEE (2005)
5. Quattoni, A., Torralba, A.: Recognizing indoor scenes (2009)
6. Galindo, C., Saffiotti, A., Coradeschi, S., Buschka, P., Fernández-Madrigal, J.A., González, J.: Multi-hierarchical semantic maps for mobile robotics. In: 2005 IEEE/RSJ International Conference on Intelligent Robots and Systems, (IROS 2005), pp. 2278–2283. IEEE (2005)
7. Gross, H.M., Boehme, H., Schroeter, C., Mueller, S., Koenig, A., Einhorn, E., Martin, C., Merten, M., Bley, A.: TOOMAS: Interactive Shopping Guide robots in everyday use-final implementation and experiences from long-term field trials. In: IEEE/RSJ International Conference on Intelligent Robots and Systems, IROS 2009, pp. 2005–2012. IEEE (2009)

8. Maier, W., Steinbach, E.: A probabilistic appearance representation and its application to surprise detection in cognitive robots. *IEEE Transactions on Autonomous Mental Development* 2(4), 267–281 (2010)
9. Boutell, M.R., Luo, J., Brown, C.M.: Factor Graphs for Region-based Whole-scene Classification. In: *Proceedings of the 2006 Conference on Computer Vision and Pattern Recognition Workshop*, page 104. IEEE Computer Society (2006)
10. Zhang, R., Zhang, Z., Li, M., Ma, W.Y., Zhang, H.J.: A probabilistic semantic model for image annotation and multi-modal image retrieval. *Multimedia Systems* 12(1), 27–33 (2006)
11. Markou, M., Singh, S.: Novelty detection: a review—part 1: statistical approaches. *Signal Processing* 83(12), 2481–2497 (2003)
12. Bishop, C.M.: Novelty detection and neural network validation. *IEE Proc.-Vls. Image Signal Process* 141(4), 217 (1994)
13. Lauritzen, S.L., Richardson, T.S.: Chain graph models and their causal interpretations. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)* 64(3), 321–348 (2002)
14. Kschischang, F.R., Frey, B.J., Loeliger, H.A.: Factor graphs and the sum-product algorithm. *IEEE Transactions on Information Theory* 47(2), 498–519 (2001)
15. Mooij, J.M.: libDAI: A free and open source C++ library for discrete approximate inference in graphical models. *Journal of Machine Learning Research* 11, 2169–2173 (2010)
16. Chow, C.: On optimum recognition error and reject tradeoff. *IEEE Transactions on Information Theory* 16(1), 41–46 (1970)